Long-Period Theory of Household Demand

Ian Steedman

ICOAE 2017, Coventry, U.K., 6~8th July

ABSTRACT

In conventional household demand theory, income and prices are varied one at a time to examine the effects on the quantities demanded. But in the long-period prices are interdependent and it makes far more sense to vary only the exogenously given prices, allowing all other prices (and incomes) to adjust accordingly.

INTRODUCTION

Inspired by Sraffa's classic *Production of Commodities* (1960), many economists have examined long-period positions in which, by definition, price equals unit cost in every industry both before and after any change in an exogenous variable. Many important results have been obtained concerning the choice of production methods, the relations between input use and input price, etc. etc.. On the other hand, the last half-century has seen little work on the implications of adopting a long-period perspective for household demand theory. Yet it is no less true for the theory of the household than for that of the firm that it makes a great difference when it is recognized that prices cannot vary independently,simply because they are interconnected via the 'price equals unit cost' equations. The purpose of this short paper is to indicate how one might begin to construct a long-period theory of household demand. (To avoid any possible misunderstanding, we emphasize that 'long-period' here refers *only* to the fact that we take account of long-run 'price equals unit cost' relationships; we make no reference to the - no doubt intriguing - dependence of current demands on past consumption experience).

We aim here at simplicity rather than generality and to that end we begin by assuming just two commodities, two primary inputs (homogeneous labour and homogeneous land), a zero rate of interest and constant-returns-to-scale in production. Only the primary input prices - the wage rate and the rent rate - can be treated as parameters; product prices and household income will be *determined by* the wage and rent rates.

We present our initial argument in two steps, first examining how the household's budget constraint varies as the wage and rent change and only then, secondly, introducing the conventional concept of a household preference ordering. (Thus even readers who shy away from utility functions, etc., may find the discussion of the budget set useful.) Some generalizations will follow.

THE BUDGET CONSTRAINT

There is no need to explain the notation used in writing the household's budget constraint as

$$p_1x_1 + p_2x_2 = e$$

But here, in a long-period framework, we suppose that $p_1 = c_1(w,r)$ and $p_2 = c_2(w,r)$, where the $c_j(\ )$ are unit cost functions, while w and r represent the wage rate and the rent rate. For now, we suppose also that the household's income/expenditure, e, is given by $e = lw + tr$, where l and t are the household's *fixed* endowments of labour and of land. If all income is spent on purchasing the two commodities, the budget constraint becomes

$$c_1(w,r)x_1 + c_2(w,r)x_2 = (lw + tr) \qquad (1)$$

Since the $c_j(\ )$ functions are linear-homogeneous in w and r, it is clear that (1) can be rewritten in terms of the *single* variable $z \equiv (r/w)$. The rent-wage ratio is the *only* variable involved in determining the budget constraint.

In a diagrammatic version of (1), with x_2 on the vertical axis and x_1 on the horizontal one, the (absolute) slope of the constraint will be $[c_1(w,r)/c_2(w,r)]$. If we assume that, for relevant (r/w), commodity 1 is always the relatively land-intensive commodity then this slope will increase monotonically as (r/w) rises. (But see below on factor-intensity-reversal.)

How will the intercepts, $X_j \equiv (lw + tr)/c_j(w,r)$, change as (r/w) rises? It is easy to show that X_j will rise if and only if $(t/l) > (t_j/l_j)$, where t_j and l_j are the inputs of land and of labour per unit of output. (They are both *variables*, of course.) Let us make the conventional assumption that, as (r/w) rises, each (t_j/l_j) falls from an indefinitely high value to zero; we have already stipulated that $(t_1/l_1) > (t_2/l_2)$ for all (r/w). It follows that, with $tl > 0,$ as (r/w) rises without limit from zero:

i) at first, both X_1 and X_2 will fall;

ii) then, for a certain range of (r/w) values, X_1 will continue to fall but X_2 will increase;

iii) above a certain (r/w), both X_1 and X_2 will increase.

We see, then, that the movement of the budget constraint, as (r/w) varies, is relatively complicated, the only monotonic movement involved being that of the (absolute) slope. Since each X_j first falls and then rises, it is natural to wonder whether it is greater at $r=0$ or at $w=0$ but unfortunately there is no general answer to that question; it depends on the properties of the $c_j(w,r)$ function.

If we now allow factor-intensity-reversal to occur, even the monotonic behaviour of the slope is lost. If we set aside the fluke case that (t/l) is exactly equal to a factor-intensity- reversing value of (t_j/l_j), it will still be true that, as (r/w) rises, we have three successive phases in which (i) both X_j fall, (ii) one X_j falls and the other increases, (iii) both X_j increase. But now the slope of the budget constraint will vary non-monotonically as (r/w) rises. Even in our very simple two good, two primary input case, changes in the rent/wage ratio can have somewhat complicated effects on the household's budget constraint.

PREFERENCE ORDERINGS AND HOUSEHOLD DEMANDS

No matter what one assumes about household preferences, the fact that the budget constraint depends on only one exogenous variable, (r/w), means that the quantities demanded will so depend:

$$x_1 = x_1(r/w) \quad \text{and} \quad x_2 = x_2(r/w)$$

That one cannot expect to say very much *a priori* about the properties of such demand functions can be seen by considering two households, with the same (l,t), each of which consumes the two commodities in fixed proportions; for one household (x_2/x_1) is 'very high' and for the other it is 'very low'. Now consider an increase in (r/w) that, as discussed above, lowers X_1 but raises X_2. The first household will respond to the increased (r/w) by *increasing both* x_1 and x_2, while the second household will respond by *decreasing both* x_1 and x_2. It is clear, then, that definite results will not easily be come by!

Consider now a household with 'Cobb-Douglas' preferences; each x_j will stand in a fixed proportion to the corresponding X_j. But it then follows from what was shown in the previous section that each $x_j(r/w)$ will be a non-monotonic relationship. Even with Cobb-Douglas preferences for every household, therefore, the aggregate demand for a commodity can be a non-monotonic function of (r/w). This will not be convenient for any theorist seeking a rent-wage ratio that gives a unique and stable equilibrium.

A less humdrum example of a possible utility index is the quadratic function

$$u = \underline{a}.\,\underline{x} - \tfrac{1}{2}(\underline{x}^T A\,\underline{x})$$

where $\underline{a} > \underline{0}$ and A is a symmetric, positive definite matrix. It follows that

$$\underline{x} = \underline{x}^* - \left(\frac{\underline{p}.\underline{x}^* - e}{\underline{p}A^{-1}\underline{p}^T}\right)A^{-1}\underline{p}^T \qquad (2)$$

where $\underline{x}^* \equiv A^{-1}\underline{a}^T$ is the *satiation* consumption bundle. Result (2) holds good for any number of commodities, of course, but even if we set $e = (lw + tr)$ and $p_j = (l_j w + t_j r)$ (with *fixed* input coefficients l_j and t_j, for $j = 1, 2$), it shows that each $(x_j^* - x_j)$ is the ratio between two quadratic expressions in (w, r). And if we introduce more general unit cost functions, $p_j = c_j(w, r)$, then our $x_j(r/w)$ can become quite complicated relationships – this with just two goods and two primary inputs.

GENERALIZATION

We now reconsider the above argument in rather more general form, there being n produced commodities with prices $(p_1, p_2, ..., p_n)$ and m primary inputs with prices $(w_1, w_2, ... w_m)$. The household is taken to maximize $u(\underline{x})$ subject to $\underline{p}\underline{x} = e$ (in obvious notation). Generalizing the above we set $e = \underline{w}.\underline{l}$, where column vector $\underline{l}$ is the household's endowment of primary inputs. (It would of course be possible to set $e = \bar{e} + \underline{w}.\underline{l} + \underline{p}.\underline{z}$, where $\bar{e}$ is exogenous income and $\underline{z}$ the household's endowment of commodities but this extension is left to the interested reader.) More important for our long-period focus is that we have

$$p_j = u_j(\underline{p}, \underline{w}) \qquad (3)$$

for $j = 1, ..., n$, where $u_j(\ )$ is a unit cost function. We may suppose that (3) can be solved to give

$$p_j = c_j(\underline{w}) \qquad (4)$$

for $j = 1, ..., n$.

As is well-known, the comparative statics of the household's maximizing decision are given by

$$\begin{bmatrix} u_{ij} & -\underline{p}^T \\ \underline{p} & 0 \end{bmatrix} \begin{pmatrix} d\underline{x} \\ d\lambda \end{pmatrix} = \begin{pmatrix} \lambda \underline{dp}^T \\ \underline{dp}.\underline{x} - de \end{pmatrix} \qquad (5)$$

where λ is the 'marginal utility of income'. Now (5) still holds in our long-period context, *provided that* we always take account of (4) and of $e = \underline{w}.\underline{l}$. The standard analysis of (5) proceeds, of course, by taking *only one element* of $\underline{dp}$ to be different from zero. But when (4) is always enforced, this may very well be an impossible procedure. If the $(i, j)th$ element of the (variable) matrix L is the use of primary input *i* per unit of output *j*, then (4) in differential form is

$$\underline{dp} = \underline{dw}L \qquad (6)$$

If $n > m$ then, flukes aside, there will be no $\underline{dw}$ vector satisfying (6) when only one element of $\underline{dp}$ is non-zero. Hence the standard argument cannot be used in the long-period context if $n > m$. It will then be far better to treat the $\underline{w}$ as our exogenous prices and to re-write the $r.h.s.$ of (5) as

$$\begin{pmatrix} \lambda L^T \underline{dw}^T \\ \underline{dw} L \underline{x} - \underline{dw}\,\underline{l} \end{pmatrix} \qquad (7)$$

Since each household demand, x_i, will be homogeneous of degree zero in $(\underline{p},e)$, it will also be homogeneous of degree zero when considered as $x_i(\underline{w})$. The standard comparative statics result (5), as modified by (7), can then be used to examine some of the properties of the $x_i(\underline{w})$ function. (It should be recalled that $du = 0$ when $\underline{dp}\,\underline{x} = de$ and thus, with reference to the last element in (7), when $\underline{dw} L \underline{x} = \underline{dw}\underline{l}$; this defines the changes in primary input prices that are consistent with a constant maximum value of the utility index.) We can easily show that

$$\left[\frac{\partial x_i}{\partial w_j}\right]_{\bar{u}} = \left[\frac{\partial x_i}{\partial w_j}\right] + \left[L\underline{x} - \underline{l}\right]. \left(\frac{\partial \underline{x}}{\partial e}\right) \qquad (8)$$

Result (8) is, clearly, reminiscent of the familiar Slutsky matrix but one must not exaggerate the similarity. The matrix on the $l.h.s.$ of (8) is *not* symmetric and negative semi-definite – and not only because it will usually not be square! Even if $n = m$, $(\partial x_i / \partial w_j)$ and $(\partial x_j / \partial w_i)$ are not of the same dimensions, so that it would have no significance to speak of their (non-)equality. (By contrast, in the Slutsky matrix $(\partial x_i / \partial p_j)$ and $(\partial x_j / \partial p_i)$ *are* of the same dimensions, so that it is meaningful to assert that they are equal in magnitude.) Note that, even if we take all commodities to be ‘normal’, so that $(\partial \underline{x} / \partial e) > \underline{0}$, the vector $(L\underline{x} - \underline{l})$ is dependent on $\underline{w}$ and can be of a variable sign-pattern, so that we do not know whether the positive $(\partial \underline{x} / \partial e)$ terms add to or subtract from the $r.h.s.$ of (8). Apart from the fact that both $\underline{x}(\underline{w})$ and $\underline{x}(\underline{w})_{\bar{u}}$ are homogeneous of degree zero, we know little about them. Summing across households to obtain market-level long-period demand curves, $\sum_h \underline{x}^h(\underline{w})$, will not increase our knowledge. Indeed, since variation in relative primary input prices will *both* provoke complicated demand changes in each household *and* change the distribution of income between households, we cannot expect to say much about market- level demand curves.

A SMALL OPEN ECONOMY

The arguments presented thus far may perhaps give the impression that our emphasis on long-period comparisons is necessarily related to taking primary input prices as the exogenous variables. That any such impression is false will now be brought out by switching our attention to a small *open* economy (and how many closed economies are there?). Once again, we do not aim at maximum generality but, rather, consider a simple case that puts our long-period emphasis sharply into focus.

Let the household maximize $u(\underline{x}^d, \underline{x})$ subject to $(\underline{p}.\underline{x}^d + \underline{\pi}\,\underline{x} = \underline{w}\,\underline{l})$, where $\underline{x}^d$ is a vector of domestic goods, $\underline{x}$ is a vector of internationally traded goods all of which are produced domestically and $\underline{\pi}$ is the exogenously given price vector of these latter goods, expressed in domestic currency. In the long-period, the prices $(\underline{p}, \underline{\pi}, \underline{w})$ are related by

$$\underline{p} = \underline{u}^d(\underline{p}, \underline{\pi}, \underline{w}) \tag{9}$$

and

$$\underline{\pi} = \underline{u}(\underline{p}, \underline{\pi}, \underline{w}) \tag{10}$$

where $\underline{u}^d(\ )$ and $\underline{u}(\ )$ are vectors of unit cost functions. Solving (9) as $\underline{p} = \underline{c}(\underline{\pi}, \underline{w})$ and then substituting into (10) we have

$$\underline{\pi} = \underline{u}[\underline{c}(\underline{\pi}, \underline{w}), \underline{\pi}, \underline{w}] \tag{11}$$

Simply in order to stress that our argument need not turn around primary input prices, we now assume that the number of domestically employed primary inputs is equal to the number of exported or import-competing goods and that (11) can be solved as $\underline{w} = \underline{w}(\underline{\pi})$.[1] In the long-period, then, the household maximizes $u(\underline{x}^d, \underline{x})$ subject to

$$\underline{p}(\underline{\pi})\underline{x}^d + \underline{\pi}\,\underline{x} = \underline{w}(\underline{\pi})\underline{l} \tag{12}$$

and the *only* prices entering the budget constraint (12) are the international prices. Needless to say, that constraint is unaffected by a proportional change in all the elements of $\underline{\pi}$. It is to be noted that $(w_1/\pi_j, \ldots w_m/\pi_j)$ cannot *all* increase (or decrease) as $\underline{\pi}$ varies and that, as a result, $\pi_j^{-1}(\underline{w}\,\underline{l})$ may change non-monotonically as π_j changes; the movement of the budget constraint can be complicated. And, of course, one *cannot* change π_j without changing $\underline{p}$. Nevertheless, the comparative statics relation (5) can now be adapted to the case of the long-period in a small open economy.

CONCLUDING REMARKS

The reader can no doubt think of many ways in which our argument could be further extended. One could introduce characteristics as well as commodities; one could allow explicitly for variable labour supply; the concepts of compensating and equivalent variations and of consumer price indexes could be adapted as necessary; one could acknowledge the role of time use and time constraints in household consumption; one could allow for a positive and variable rate of interest, *i*, both in production – so that $p_j = c_j(\underline{w}, i)$ – and in influencing household intertemporal consumption decisions.[2] And so on. Whatever extention is pursued, however, the central point of long-period theory will remain the same; it is not meaningful to pretend to vary prices one at a time, because prices are interdependent.[3]

It would be a mistake to suppose that Sraffa's (1960) emphasis on the role of 'price equals cost' relations is of importance *only* in the context of production theory, or *only* in the presence of a positive and variable rate of interest. Such long-period relations can make a big difference to economic theorizing even when the rate of interest is zero (c.f., Opocher and Steedman, 2015) and even, it has been argued here, when attention is turned to household demand theory. Much work still remains to be done in the process of drawing out all the implications of Sraffa's emphasis on long-period positions.

REFERENCES

Opocher, A. and Steedman, I. (2015), *Full Industry Equilibrium. A Theory of the Industrial Long-Run*. Cambridge University Press.

Sraffa, P. (1960), *Production of Commodities by Means of Commodities*. Cambridge University Press.

Steedman, I. (2016), On the Marginalist Theory of Capital Supply. *Metroeconomica*, vol.67 (1), pp. 114-118

Footnote 1 True, we could also solve as $\underline{\pi} = \underline{\pi}(\underline{w})$ and then (12) would be indistinguishable from our earlier budget constraint in terms of $\underline{w}$. But it remains that $\underline{w}$ is not the inevitable focus of attention. And if there are *more* primary inputs than traded goods, it may be of interest to take $\underline{\pi}$ and *some* w_j to be exogenous.

Footnote 2 An initial step in this direction was taken in Steedman (2016); the reader might wish to merge that paper's argument with the themes of the present one.

Footnote 3 This is a central theme of Opocher and Steedman (2015, *passim*). That work also points out that if each industry consists of identical firms with U-shaped cost curves, then it is often possible to replace constant-returns-to scale and unit cost functions by the condition that price equals minimum average cost, industry by industry. That could have been done throughout the present paper.