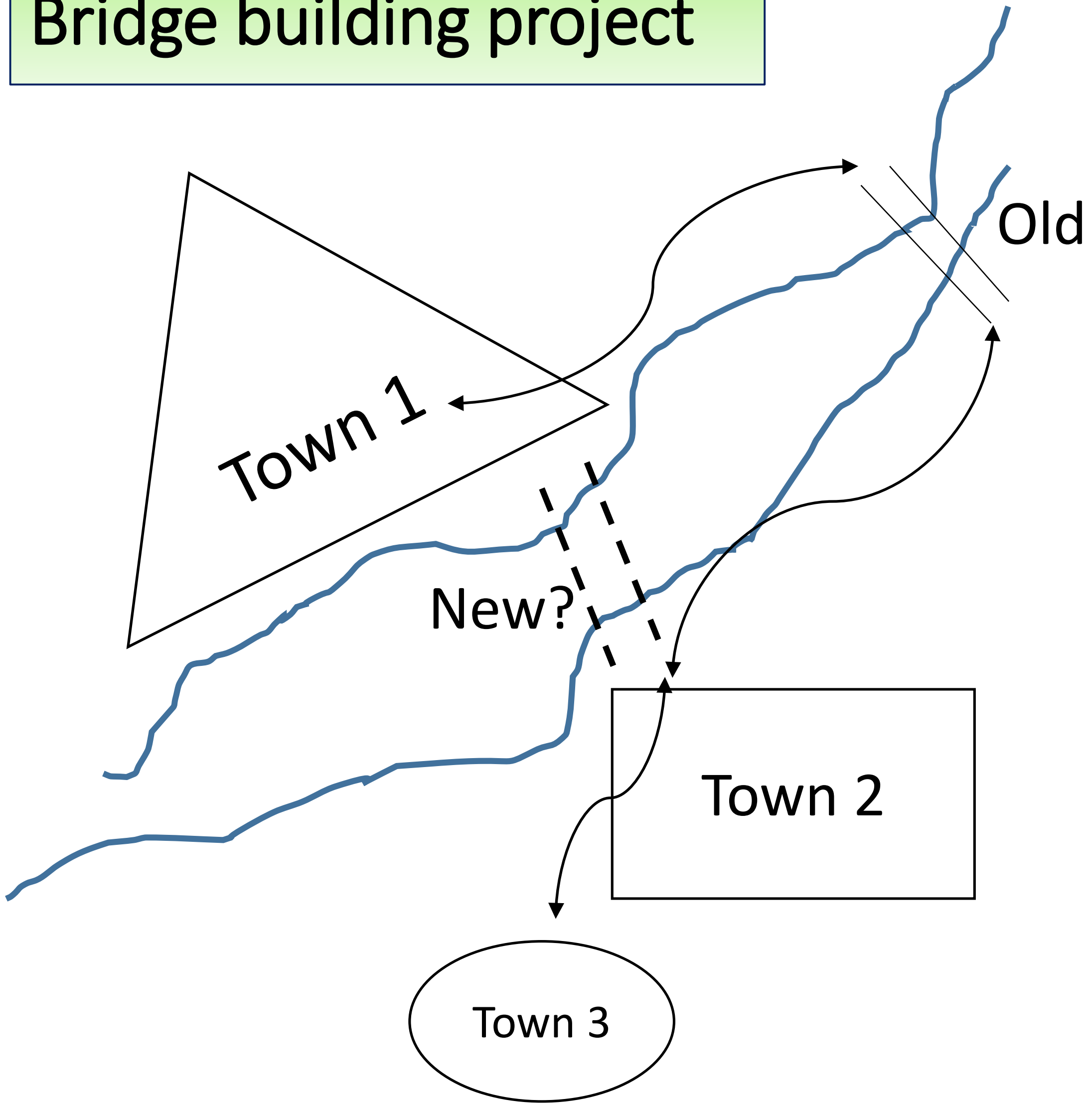
Majority Decision Rules with Minority Protections: Cost Assignments for Public Projects

by M. Kaneko, July 7, Warsaw (based with a paper with Ryohei Shimoda)

- Majority decision making is basic for democracy.
  - It collects diversified knowledge, opinions, and desires in a society, in order to
    - Provide decisions to the society;
    - Treat all participants equally.
  - However, it has various defects:
    - It may provide no stable decision and the society may be chaotic;
    - It may create tyranny of a majority with exploitations of a minority.
  - Majority decision for democracy was discussed by Tocqueville (1835), Kelsen (1929).
  - It is used at many levels of communities, towns, states, nations, . . .
- This paper considers majority decision rules with minority protections in the context of cost assignments for public projects.

Bridge building project

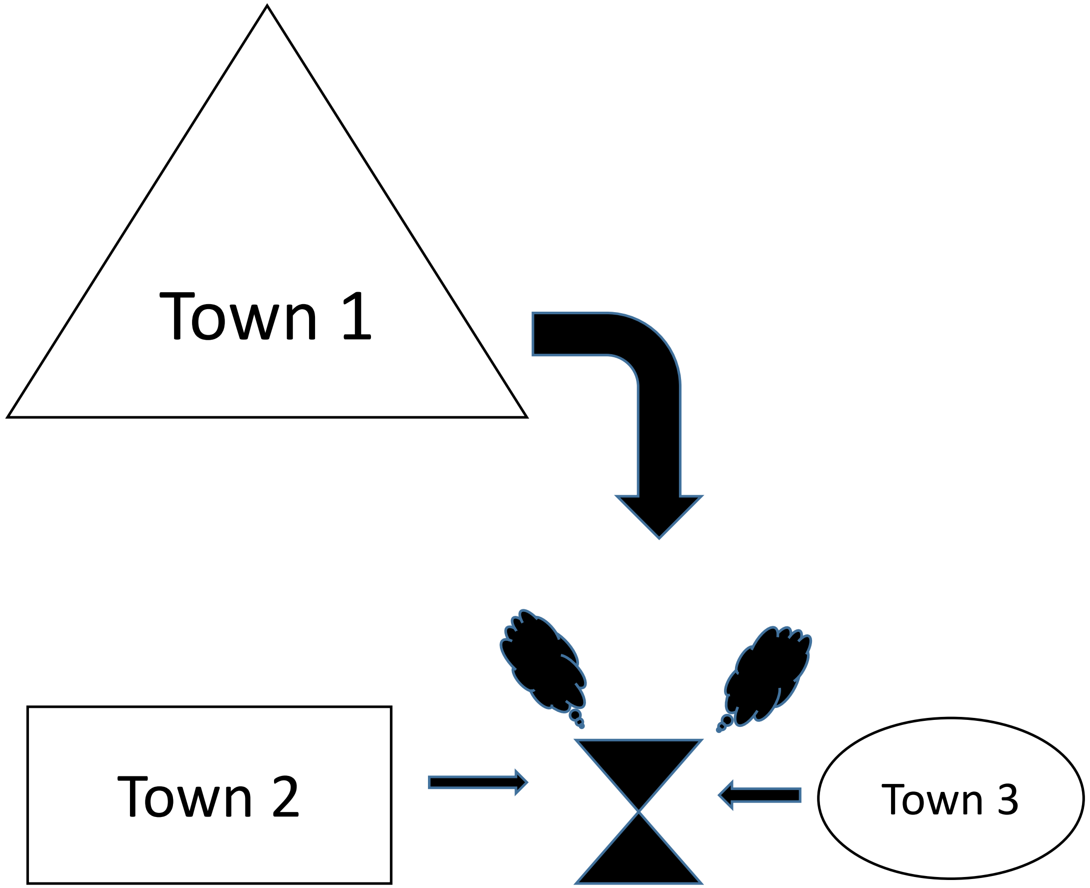


1. Three neighboring towns are facing a river.
2. There is an old bridge, located far from these towns.
3. They decide to build a new bridge.
4. The river is “public” in the sense that building needs the consensus of all the towns.
5. The problem is how to allocate the total costs (and/or total surpluses).
6. Each has the right to refuse the project. If one refuses, the project is invalid.

E.g., benefits for them: $b_1(1) = 1$, $b_2(1) = \frac{4}{5}$, $b_3(1) = \frac{1}{5}$, $C(1) = 1$.

➢ **Normalization**: $b_i(0) = 0$ for $i = 1,2,3$.

Waste Incineration Project



1. Three neighboring towns are building a waste incineration plant.
2. Town 1 has large benefits from it, but towns 2, 3 will suffer from environmental problems.
3. Assume $\sum_{i=1}^{3} b_i(1) - C(1) > 0$

   ➔ $b_1(1) > C(1)$;

   $b_2(1) < b_2(0) = 0$;

   $b_3(1) < b_3(0) = 0$.

E.g., $b_1(1) = 3$, $b_2(1) = b_3(1) = \frac{-1}{2}$, and $C(1) = 1$.

What are defects of the majority decision rules?

Two different viewpoints

- ***Ex ante*** - - as rules of the game,
  - the society as a whole makes decisions for themselves;
  - all participants should be treated equally with equally granted rights;
    - The simple majority decision rule satisfies these requirements.
- ***Ex post - -*** for long-run,
  - the society may be chaotic;
  - otherwise, it may create tyranny of a majority.
    - The simple majority decision rule exhibits these - - next slide for some studies.
- Is any right granted to a minority? Or can a majority ignore the minority?

Related literature in economics and game theory

- Condorcet Cycle (18th C.)
- Arrow (1951)
- Von Neumann-Morgenstern (1944)
  - Rules of the majority decision making - - cooperative game.
  - Individual rights, to refuse the project.
  - Theory of stable sets - - a possibility of studying tyranny of majority at the behavioral level.

- Kaneko (1977a, b) - - (Lindahl-) Ratio equilibria and the core of a voting game for a public good economy;
  - rules of majority decision making includes minority protection,
    in addition to individual rights.

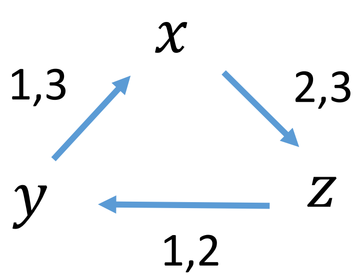
Condorcet Cycle

- Three voters 1, 2, 3
- Three social alternatives with preferences:

  1: $x \succ_1 y \succ_1 z$

  2: $y \succ_2 z \succ_2 x$

  3: $z \succ_3 x \succ_3 y$



- Majority decision rule leads to no stable outcome.

- Where are individual rights?
  - Implicit (hidden) or nothing.

Arrow's (1951) Impossibility Theorem

- Five plausible requirements for democratic decision rules are logically inconsistent.

Specifically,

- Dictatorship is implied by the other four requirements.

- It means that the social decision rule as an institution may go to dictatorship.

- This result implies neither of
  - chaos led by the majority decision rule;
  - tyranny of a majority.

Neumann-Morgenstern's (1944) simple majority game

3-player simple majority game is given as

- $v(S) = \begin{cases} 1 & \text{if } |S| \geq 2 \\ 0 & \text{if } |S| \leq 1. \end{cases}$
- The total surplus, 1, is distributed among the three players
- Public project: the total surplus $\sum_{i=1}^{3} b_i(1) - C(1) > 0$.

$$v(S) = \begin{cases} \sum_{i=1}^{3} b_i(1) - C(1) & \text{if } |S| \geq 2 \\ 0 & \text{if } |S| \leq 1. \end{cases}$$

- Bridge project: $b_1(1) = 1$, $b_2(1) = \frac{4}{5}$, $b_3(1) = \frac{1}{5}$, and $C(1) = 1$ ➔ $v(S) = \begin{cases} 1 & \text{if } |S| \geq 2 \\ 0 & \text{if } |S| \leq 1. \end{cases}$
- Incineration Plant: $b_1(1) = 3$, $b_2(1) = b_3(1) = \frac{-1}{2}$, and $C(1) = 1$ ➔ the same.

➢ **The derived game is symmetric, but the basic situation is not symmetric.**

➢ **An analysis by payoffs is not enough; we should look at the basic situation.**

Bridge project: $b_1(1) = 1$, $b_2(1) = \frac{4}{5}$, $b_3(1) = \frac{1}{5}$, and $C(1) = 1$.

- The core is empty: consider a payoff vector $a = \left(\frac{10}{15}, \frac{3}{15}, \frac{2}{15}\right)$; let $b = \left(\frac{3}{15}, \frac{2}{15}, \frac{10}{15}\right)$, $c = \left(\frac{2}{15}, \frac{10}{15}, \frac{3}{15}\right)$. Then, a, b, c form a Condorcet cycle.
- Rules of the game includes "tyranny of a majority".
- A majority S can choose any allocation of the total surplus.

Constraint: $v(\{i\}) = 0$ describes the individual right i.e., to refuse the project .

- The symmetric cost al. $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ violates this, i.e., $\frac{1}{5} - \frac{1}{3} < 0 = v(\{i\}) = 0$.
- The symmetric payoff vector $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ comes from the cost al. $\left(\frac{10}{15}, \frac{7}{15}, -\frac{2}{15}\right)$;
  - town 3 receives a subsidy.
  - this payoff vector is not reached only by allocating costs, but needs a transfer of benefits.

Minority protection in the majority decision rule

The majority game with a ratio vector:

- a (cost-assignment) ratio vector $r = (r_1, r_2, r_3)$ $with$ $r_1 + r_2 + r_3 = 1.$
- $v_r(S) = \begin{cases} max[0, \ \sum_{i\in S} u_i(1) - \sum_{i\in S} r_i C(1))] & \text{if } |S| \geq 2 \\ 0 & \text{if } |S| \leq 1. \end{cases}$
- a majority coalition $S = \{i, j\}$ obtains the total net surplus for S by paying the total costs imposed by the ratios r_i and r_j on them.

- Three minority protections:
  - **Right to refuse**: Each k he has the right to refuse the project - - $v_r(\{k\}) = 0.$
  - **Minority protection**: When majority $S = \{i, j\}$ decides to take the public project, the minority $\{k\} = N - S$ should pay at most the cost imposed by the ratio r_k.
  - **No surplus-exploitation by majority**: No transfer of surpluses from $\{k\}$ to $S = \{i, j\}$.

Bridge Project: $u_1(1) = 1 = \frac{5}{5}, u_2(1) = \frac{4}{5}, \ u_3(1) = \frac{1}{5}$, and $C(1) = 1$.

1. $r = (r_1, r_2, r_3) = (\frac{5}{10}, \frac{4}{10}, \frac{1}{10})$ (benefit proportional)
   - The game (N, v_r) has the unique core payoff vector $(\frac{5}{10}, \frac{4}{10}, \frac{1}{10})$.
2. $r = (r_1, r_2, r_3) = (\frac{10}{15}, \frac{7}{15}, \frac{-2}{15})$ (3 receives a subsidy) ➔
   - The game (N, v_r) has the unique core payoff vector $(\frac{5}{15}, \frac{5}{15}, \frac{5}{15})$.
3. $r = (r_1, r_2, r_3) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ (equal ratios) ➔ The core of (N, v_r) is empty.
   - Town 3 refuses the project, since his cost al. is too much, i.e., $\frac{1}{5} - \frac{1}{3} < 0$.

- Q1: When is the core of (N, v_r) noonempty or empty?
- Q2: What is the structure of the core if it is nonempty?

Core of (N, v_r) with minority protection $r = (r_1, r_2, r_3)$: $\sum_{i \in N} r_i = 1$.

Theorem A: For any benefits $(b_1(1), b_2(1), b_3(1))$, there is a ratio vector $r = (r_1, r_2, r_3)$ such that (N, v_r) has the nonempty core.

- Assume $\sum_{i \in N} b_i(1) - C(1) > 0$ but not necessarily $b_i(1) > 0$ for $i \in N$.

Theorem B: (N, v_r) has the nonempty core $\Leftrightarrow$ $\frac{b_i(1)}{C(1)} \geq r_i$ for $i = 1,2,3$.

- When $b_i(1) < 0$, we have $r_i < 0$; i should receive subsidy $r_i C(1)$.

- everybody wants to have the public project since $b_i(1) - r_i C(1) \geq 0$ for $i = 1,2,3$.

Theorem C: (N, v_r) has the nonempty core $\Leftrightarrow$ the core consists of a unique payoff vector $[b_i(1) - r_i C(1): i = 1,2,3]$ and $b_i(1) - r_i C(1) \geq 0$ for $i = 1,2,3$.

- Player i should pay the cost assigned by the ratio r_i for the project.

Define $r_i = \frac{b_i(1)}{\sum_{j \in N} b_j(1)}$ for $i \in N$ (benefit proportional).

- When $b_i(1) > 0$ for $i \in N$, (N, v_r) with the b-p $r = (r_1, r_2\ r_3)$ has the nonempty core.

- **Incineration Plant**: $b_1(1) = 3$, $b_2(1) = b_3(1) = \frac{-1}{2}$, and $C(1) = 1$ ➔
  - The b-p ratio vector is $\left(\frac{6}{4}, \frac{-1}{4}, \frac{-1}{4}\right)$ and the corresponding payoff vector is $(\frac{6}{4}, \frac{-1}{4}, \frac{-1}{4})$. The core is empty.
  - When the ratio vector is $\left(\frac{4}{2}, \frac{-1}{2}, \frac{-1}{2}\right)$ and the corresponding payoff vector is $(1,0,0)$, which is the core payoff vector.

Uniform benefits for the people in the three towns

Suppose

- $b_i(1) > 0$ for $i \in N$;
- n_i is the population of town i and $b > 0$ is the benefit for each person in town i;
- $b_i(1)$ is the sum of benefits of the population of town i, i.e., $b_i(1) = n_i b$.

Theorem D: Let $r = (r_1, r_2, r_3)$ be benefit-proportional, and (x_1, x_2, x_3) the core vector of (N, v_r). Then,

1. (equal net benefits for people): $\frac{x_1}{n_1} = \frac{x_2}{n_2} = \frac{x_3}{n_3}$;

2. (equal cost sharing for people): $\frac{r_1 C(1)}{n_1} = \frac{r_2 C(1)}{n_2} = \frac{r_3 C(1)}{n_3}$.

Comparisons of the three approaches

- vN-M simple game

$$v(S) = \begin{cases} \sum_{i=1}^{3} b_i(1) - C(1) & \text{if } |S| \geq 2 \\ 0 & \text{if } |S| \leq 1. \end{cases}$$

- Majority game with minority protections

$$v_r(S) = \begin{cases} max[0, \sum_{i \in S} u_i(1) - \sum_{i \in S} r_i C(1))] & \text{if } |S| \geq 2 \\ 0 & \text{if } |S| \leq 1. \end{cases}$$

- Market game approach:

$$v_m(S) = max[0, \sum_{i \in S} u_i(1) - C(1)] \quad \text{for all } S \subseteq N.$$

- This is a direct application of the "market game" (see Foley ('70)).
- The game is a convex game; the core is very large.
- The "public project" has a different meaning; each coalition can construct a project such as a bridge, excluding the other town.

More analysis

- The study presented is closely related to:
  - Lindhal equilibrium - - Lindhal (1919) (cf., van den Nouweland (2015))
  - (Lindhal) Ratio equilibrium - - Kaneko (1977a, b)
    - r is an equilibrium ratio $\Leftrightarrow$ (N, v_r) has the nonempty core.
    - When r is an equilibrium ratio vector, the ratio equilibrium is the core al. of (N, v_r).

- Endogenous determination of a ratio vector, through the majority decision game.

- Drop the quasi-linearity assumption on preferences for the participants.
  - Income effects can be incorporated (cf., Kaneko ('17)).
  - The problem of "rich vs. poor" can be studied.

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